11. Flow Measurements

1. Flow rate measurement

A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.

\[ Q = \frac{\text{Volume}}{\text{Time}} \]

2. Velocity measurement

(1) Pitot probe (or Pitot tube)

\[ h = \frac{V^2}{2g} \]

Stagnation pressure head
(2) Pitot-static probe (皮托靜壓管)

Pitot formula:

\[ V = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \]
(3) Current meter (流速儀)

\[ V = a + bN \]

where \( V \) = current velocity (m/s)
\( N \) = rotation (rpm)

(4) Obstruction flowmeters (阻流式流速儀)

Flow through a constriction in a pipe

**Mass balance:** \( A_1 V_1 = A_2 V_2 \rightarrow V_1 = \frac{A_2}{A_1} V_2 = \left( \frac{d}{D} \right)^2 V_2 \) \hspace{1cm} (1)

**Bernoulli equation** (\( z_1 = z_2 \)): \[ \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \] \hspace{1cm} (2)

Combining Eqs. (1) and (2) and solving for velocity \( V_2 \) gives

**Obstruction (with no loss):** \( V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^2)}} \)

where \( \beta = \frac{d}{D} \)
\[ Q = A_2 V_2 = \frac{\pi d^2}{4} V_2 \]
流量係數

\[ Q = A_d C_d V_2 \]

where \( C_d \) = 流量係數

For \( Re > 30,000 \)

- \( C_d = 0.96 \) (Nozzle)
- \( C_d = 0.061 \) (Orifice)
- \( C_d = 0.98 \) (Venturi meter)
(5) Sonic (or Acoustic) flowmeter (聲波式流速儀)

- Transit time sonic flowmeter (傳递时差式)

\[ V = KL\Delta t \]

where
\[ K = \text{a constant} \]
\[ L = \text{distance between transducers} \]
\[ \Delta t = \text{travel time difference} \]


- Doppler-effect (Frequency shift) sonic flowmeter (杜卜勒效應式)

The operation of a Doppler-effect ultrasonic flowmeter equipped with a transducer pressed on the outer surface of a pipe.
King’s law:

\[ E^2 = a + bV^n \]

where \( E \) = voltage
\( a, b, n \) = constants to be calibrated

Thermal anemometer probes with single, double, and triple sensors to measure (a) one-, (b) two-, and (c) three-dimensional velocity components simultaneously.
(7) Laser Doppler Velocimetry (LDV) or Laser Doppler Anemometry (LDA) (雷射杜卜勒流速儀)

A non-intrusive method (利用光學原理 非浸入式流速儀)

A dual-beam LDV system in forward scatter mode

\[ s = \frac{\lambda}{2 \sin(\alpha / 2)} \]

where
- \( s \) = spacing between fringe lines
- \( \lambda \) = wavelength of fringes
- \( \alpha \) = angle between two laser beams

\[ f = \frac{V}{s} = \frac{2V \sin(\alpha / 2)}{\lambda} \]

where
- \( f \) = frequency of scattered fringe lines
- \( V \) = velocity of a particle traversing fringe lines

Fringes that form as a result of the interference at the intersection of two laser beams of an LDV system (lines represent peaks of waves). The top diagram is a close-up view of two fringe lines.
(8) Particle Image Velocimetry (PIV) 粒子影像流速儀

A PIV system to study flame stabilization.
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wake region of a car as measured by a PIV system in a wind tunnel. The velocity vectors are superimposed on a contour plot of pressure. The interface between two adjacent grayscale levels is an isobar.
*Courtesy Dantec Dynamics, Inc. www.dantecmt.com.*
3D PIV system

A three-dimensional PIV system set up to study the mixing of an air jet with cross duct flow.

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